Dottorato di Ricerca in Economia Politica, XVIII ciclo

Microeconomics: production and cost functions

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Consider the production function

$$y = 1000[1 - \exp\{-\Phi_i(L, K)\}],$$

where y is the production (output), while L and K are the inputs, namely the labour and the capital. Let $w = 2r$ be the relationship between the cost of labour (wage w) and the price of capital (r).
Consider the two following cases for the function Φ_i:

- $i = 1 \;\Rightarrow\; \Phi_1(L, K) = LK$, with $L > 0$ and $K > 0$,
- $i = 2 \;\Rightarrow\; \Phi_2(L, K) = L + K$,with $L \geq 0$ and $K \geq 0$.

1. Prove that the maximum possible value of y is the same for $i = 1$ and $i = 2$, and calculate it. (4 pt)

   proof: ____

   ____

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   ____

   $y_{\max} =$ ______________

2. Compute the Marginal Rate of Technical Substitution (MRTS), in both cases $i = 1$ and $i = 2$. (4 pt)

   $|\text{MRTS}_1(L, K)| =$ ______________ $|\text{MRTS}_2(L, K)| =$ ______________

3. In what case the perfect substitution between inputs L and K exists? (2 pt)

   ○ only when $i = 1$,
   ○ only when $i = 2$,
   ○ in both cases,
   ○ never.

4. In what case the elasticity of substitution is the same as the Cobb-Douglas function? (4 pt)

   ○ only when $i = 1$,

- ◯ only when $i = 2$,
- ◯ in both cases,
- ◯ never.

5. Compute the elasticity of the production y with respect to the capital input when $i = 2$. (4pt)

   $\varepsilon_{y,K} =$ ________________________

6. Compute the conditional demand of the inputs L and K when $i = 1$. (4pt)

   $L_1^* =$ ________________________ $K_1^* =$ ________________________

7. Compute the conditional demand of the inputs L and K when $i = 2$. (4pt)

   $L_2^* =$ ________________________ $K_2^* =$ ________________________

8. Compute the total cost function in both cases $i = 1$ and $i = 2$. (6 pt)

   $\mathrm{TC}_1 =$ ____

   $\mathrm{TC}_2 =$ ____