PhD in Economics and Social Sciences (13th Cycle)
Econometrics test (2012-10-17)

Name: ____

1. Say if the following statements are unambiguously true (TRUE), unambiguously false (FALSE) or impossible to classify the way they are stated (CAN'T SAY). Write the motivations to your answers only in the space provided. Answers with no motivations will not be considered.

   (a) In order to use asymptotic results, the data we observe must be iid.
   TRUE ○ FALSE ○ CAN'T SAY ○

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   ____
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   (b) An estimator may be biased and still be consistent.
   TRUE ○ FALSE ○ CAN'T SAY ○

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   (c) An estimator whose variance exists and has a non-zero limit as the sample grows without limit may still be consistent.
   TRUE ○ FALSE ○ CAN'T SAY ○

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   (d) Assume that $X_n \xrightarrow{\text{p}} X$ and consider the two following statements: (a) $X_n - X \xrightarrow{\text{p}} 0$ and (b) $X_n \xrightarrow{\text{d}} X$. Both (a) and (b) are true.
   TRUE ○ FALSE ○ CAN'T SAY ○

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   (e) If a model contains endogenous regressors, OLS returns an incorrect estimate of the conditional mean function.
   TRUE ○ FALSE ○ CAN'T SAY ○

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2. The following is a model for the educational level of household heads, as per the 2010 *Survey on Household Income and Wealth* (source: Bank of Italy).

The explanatory variables are:

Variable Name	Description
ETA	Age in years
SEX	Gender (1=male)
STUPCF	Educational attainment of the individual's father
PARAVG	Average attainment of the individual's parents

In other words, the variable PARAVG is the sum of the individual's father and mother education level, divided by two.

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OLS, using observations 1-7951 (n = 6571)
Missing or incomplete observations dropped: 1380
Dependent variable: STUDIO
Heteroskedasticity-robust standard errors, variant HC1

             coefficient   std. error   t-ratio     p-value
  ---------------------------------------------------------
  const       3.81654      0.126701      30.12     7.89e-187 ***
  ETA        -0.0233893    0.00132060   -17.71     1.33e-68  ***
  SEX        -0.255347     0.0339268     -7.526    5.91e-14  ***
  STUPCF      0.0467222    0.0656243      0.7120   0.4765
  PARAVG      0.765759     0.0768872      9.960    3.35e-23  ***

Mean dependent var   3.866991   S.D. dependent var   1.675581
Sum squared resid    12381.75   S.E. of regression   1.373221
R-squared            0.328748   Adjusted R-squared   0.328339
F(4, 6566)           649.8102   P-value(F)           0.000000
Log-likelihood      -11405.40   Akaike criterion     22820.80
Schwarz criterion    22854.75   Hannan-Quinn         22832.53
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(a) Estimate the effect of father's education (β_F)

(b) Estimate the effect of mother's education (β_M)

(c) Carry out a test for the hypothesis that the impact on the dependent variable of the educational attainment is equal between mother and father ($\beta_M = \beta_F$).

(d) Discuss the estimation output.

3. Suppose you have a random sample of 100 i.i.d. observations $x_1, \ldots, x_n$ from X with probability density function

$$f(x, c) = \begin{cases} cx^{c-1} & \text{for} \quad x \in [0, 1] \\ 0 & \text{elsewhere} \end{cases}$$

and

$$\sum_{i=1}^{n} \ln x_i = -80 \quad \text{and} \quad \bar{x} = \frac{5}{9}$$

(a) find a consistent estimator of c;

(b) test $H_0 : c = 1$ using the LM statistic.

4. Given the following model for the time series y_t

$$y_t = \mu + \phi y_{t-2} + \beta_0 x_t + \beta_1 x_{t-1} + \varepsilon_t, \tag{1}$$

where ε_t is a white noise with zero mean and variance σ^2. The model (1) has been estimated by using OLS and the results are

$\hat{\mu} = 1.2$ (standard error= 0.8)
$\hat{\phi}_2 = 0.65$ (standard error= 0.25)
$\hat{\beta}_0 = 0.27$ (standard error= 0.03)
$\hat{\beta}_1 = -0.08$ (standard error= 0.02)

and $Cov(\hat{\phi}, \hat{\beta}_0) = 0$, $Cov(\hat{\phi}, \hat{\beta}_1) = 0.003$, $Cov(\hat{\beta}_0, \hat{\beta}_1) = 0.005$. The estimate of the variance is $\hat{\sigma}^2 = 1$.

(a) Classify the model (1) by selecting one (or more) of the following solutions

○ ADL(1,1) ○ ADL(2,1) ○ ADL(1,2) ○ ADL(0,2)
○ ARMA(1,1) ○ ARMA(2,1) ○ ARMA(1,2) ○ ARMA(2,2)

(b) Write the null hypothesis H_0 under which the model reduces to a static model.

H_0 : ______________________________

(c) Compute a test for the above hypothesis.

Test: ____________ Distribution: ________ Test stat.: ______________
Result: ACCEPT ○ REJECT ○

(d) Write the ECM form for model (1).

$\Delta y_t =$ ____

(e) Carry out a test for the hypothesis $H_0 : \phi + \beta_0 + \beta_1 = 1$

Test: ____________ Distribution: ________ Test stat.: ______________
Result: ACCEPT ○ REJECT ○

(f) Is it possible to constrain the model in order to obtain an AR(2) model?

YES ○ NO ○ CAN'T SAY ○

If your answer is

- YES: write the implicit parameter restrictions. H_0 : ______________________
- NO or CAN'T SAY: briefly write your reasons

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Now let the process be

$$y_t = \mu + \phi y_{t-2} + \varepsilon_t \qquad (2)$$

where $\varepsilon_t \sim WN(0, \sigma^2)$ and the estimated parameters $\hat{\mu}$ and $\hat{\phi}$ are the same as the above. As you know, this process can be expressed also as $\Phi(L)y_t = \mu + \varepsilon_t$.

(a) Estimate the roots of the polynomial $\Phi(L)$.

Roots: ____

(b) Is model (2) stationary?

YES ○ NO ○ CAN'T SAY ○

(c) Compute $E(y_t)$ and $Var(y_t)$.

$E(y_t) =$ ____________________ $Var(y_t) =$ ____________________

(d) The autocorrelation of order one ($\hat{\rho}_1$) is

○ -1	○ 1	○ $\hat{\phi}_1 = 0$	○ $\hat{\phi}_2 = 0.65$
○ $\hat{\phi}_1 + \hat{\phi}_2$	○ $\hat{\phi}_1 - \hat{\phi}_2$	○ $\hat{\phi}_1/(1 - \hat{\phi}_2)$	○ not available