PhD in Economics (14th Cycle)
Econometrics test (2013-10-11)

Name: ______________________________

1. Say if the following statements are unambiguously true (TRUE), unambiguously false (FALSE) or impossible to classify the way they are stated (CAN'T SAY). Write the motivations to your answers **only** in the space provided. A "CAN'T SAY" answer with no motivations will be considered wrong.

   (a) If you estimate a model by maximum likelihood, the Hessian is a valid estimator of the covariance matrix of the parameters.
   TRUE ◯ FALSE ◯ CAN'T SAY ◯

   (b) If we perform a test by which we accept hypothesis A and another test by which we accept hypothesis B, then carrying out a further test for the joint hypothesis $A \wedge B$ is useless, since it is logically impossible that the joint hypothesis is false.
   TRUE ◯ FALSE ◯ CAN'T SAY ◯

   (c) if $X_n \xrightarrow{\text{d}} X$, then $\lim_{n\to\infty} P[X_n \in (0,1)] = P[X \in (0,1)]$.
   TRUE ◯ FALSE ◯ CAN'T SAY ◯

   (d) The OLS estimator is a Maximum Likelihood estimator.
   TRUE ◯ FALSE ◯ CAN'T SAY ◯

   (e) The OLS estimator is a GMM estimator.
   TRUE ◯ FALSE ◯ CAN'T SAY ◯

2. You have a dataset on arrests in the year 1986 and other information on 2,725 men in California. Each man is already known to the police as already arrested at least once prior to 1986. Let arr_i be a binary variable equal to 1 if individual i was arrested during 1986 and 0 if not arrested in 1986. The variable $white_i$ is a dummy variable equal to 1 if individual i is white. The variable $pcnv_i$ is the fraction of arrests prior to 1986 that led to conviction, $durat_i$ is the recent unemployment duration in months prior to 1986, and inc_i is the 1986 legal income (in $1,000). Define $\mathbf{x}_i \equiv [white_i \; pcnv_i \; durat_i \; inc_i]$ Suppose that arr_i follows a logit model

$$\Pr(arr_i = 1|\mathbf{x}_i) = \frac{\exp(\alpha + \mathbf{x}_i\boldsymbol{\beta})}{[1 + \exp(\alpha + \mathbf{x}_i\boldsymbol{\beta})]}, \tag{1}$$

where α is the constant. Table **??** reports the summary statistics of the dependent variable and of the independent variables. Table **??** displays the maximum likelihood estimation results of parameters the logit model in Eq. (**??**). Finally, Table **??** displays the 5 by 5 variance-covariance matrix of the estimated parameters.

Table 1: Summary statistics of variables

Variable	observations	mean	std. dev.	min.	max.
arr	2725	.277064	.447631	0	1.0
white	2725	.621284	.485156	0	1.0
pcnv	2725	.357787	.395192	0	1.0
durat	2725	2.251376	4.607063	0	25.0
inc	2725	5.496705	6.662721	0	54.1

Table 2: Estimation results of the logit model for being arrested in 1986

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Logit, using observations 1-2725
Dependent variable: arr
Standard errors based on Hessian
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	coefficient	std. error	z	p-value	
const	-0.0106850	0.0917962	-0.1164	0.9073	
white	-0.581706	0.0898887	-6.471	9.71e-011	***
pcnv	-0.970731	0.124498	-7.797	6.33e-015	***
durat	0.0190081	0.00940882	2.020	0.0434	**
inc	-0.0722517	0.00897769	-8.048	8.42e-016	***

Mean dependent var	0.277064	S.D. dependent var	0.447631
McFadden R-squared	????????	Adjusted R-squared	0.066102
Log-likelihood	-1496.880	Akaike criterion	3003.761
Schwarz criterion	3033.312	Hannan-Quinn	3014.442

(a) At the bottom of Table **??** the McFadden Pseudo-R-squared is not reported. Write down the formula of the Pseudo-R-Squared and, then, numerically compute it.[1]

[1] Hint 1: exploit the connection between the fraction of individuals who are arrested in 1986 and the log-likelihood value of the logit model with the constant only; Hint 2: $\ln(0.277064) = -1.2835068; \ln(0.722936) = 0.32443458$.

Table 3: The variance-covariance matrix of the estimated parameters

	cons	white	pcnv	durat	inc
cons	.00842655				
white	-.00427567	.00807997			
pcnv	-.00512757	.0002189	.01549963		
durat	-.00034955	-.00002105	.00002217	.00008853	
inc	-.00040186	-.00005257	.00010022	.00002842	.0000806

(b) Compute the predicted probability of being arrested in 1986 of individual i who is white, with $pcnv_i = 0$, no previous unemployment, and no legal income. Using the delta method, compute also the standard error of this predicted probability.

(c) Write down the analytical formulas to compute the partial effects at the average (PEA) for a continuous regressor **and** a binary regressor. Then, estimate the PEA of one more month spent in unemployment prior to 1986.

3. The Phillips Curve is a theoretical relationship between the inflation rate p_t and the unemployment rate u_t. In some of its versions, the equation

$$p_t = p_t^e + \beta(u_t - u_{t-1}), \tag{2}$$

where p_t^e is inflationary expectation at time t should be an adequate model for inflation dynamics. Under the assumption of a constant expected inflation rate, the dynamic model

$$p_t = \mu + \phi p_{t-1} + \beta_0 u_t + \beta_1 u_{t-1} + \varepsilon_t, \tag{3}$$

where $\varepsilon_t \sim WN(0, \sigma^2)$, has been estimated over a sample of monthly data ranging from 1980:1 to 2012:7; the results are in Table **??**.

Table 4: OLS, using observations 1980:01-2012:07 ($T = 391$)

Dependent variable: p

	coefficient	std. error	t-ratio	p-value	
const	0.108750	0.0509009	2.136	0.0333	**
p_1	0.574425	0.0410135	14.006	0.0000	***
u	-0.080381	0.0724048	-1.110	0.2676	
u_1	0.081738	0.0725105	1.127	0.2603	

Mean dependent var	0.278774	S.D. dependent var	0.303455
Sum squared resid	23.81405	S.E. of regression	0.248063
R-squared	0.336899	Adjusted R-squared	0.331759
F(3, 387)	65.54046	P-value(F)	2.74e-34
Log-likelihood	-7.711535	Akaike criterion	23.42307
Schwarz criterion	39.29790	Hannan-Quinn	29.71531
rho	0.023943	Durbin's h	0.806220

Breusch-Godfrey test statistic: TR^2=22.926589, p-value=0.000131
Ljung-Box test statistic: Q=11.1129, p-value=0.0253

The covariance matrix of the estimated parameters is

$$\hat{V} = \begin{bmatrix} 0.0025909 & & & \\ -0.0004362 & 0.0016821 & & \\ -0.0001260 & -0.0001300 & 0.0052425 & \\ -0.0002342 & 0.0001242 & -0.0052219 & 0.0052578 \end{bmatrix}$$

(a) Write the null hypothesis H_0 under which model (**??**) reduces to model (**??**).

H_0 : ______________________________

(**Hint:** The test-statistic returns 98.2045 and the related p-value is 0.0000.)

(b) Which is the expected sign of β in model (**??**)?

◯ POSITIVE ◯ NEGATIVE ◯ ZERO

Why? ______________________________

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(c) Write the null hypothesis H_0 under which model (**??**) reduces to an AR(1) model.

H_0 : ______________________________

(d) Carry out the test

Test:	________	Distribution:	______	Test stat.:	________
Result:	ACCEPT ◯		REJECT ◯		

(e) Fill the blanks with the estimated coefficient of the ECM form

$$\Delta p_t = \_\_\_\_ + \_\_\_\_\Delta u_t + \_\_\_\_[p_{t-1} - \_\_\_\_u_{t-1}]$$

(f) Using also the information arising from previous answers, provide some comments to the estimates contained in Table **??**.