PhD in Economics (16th Cycle)
Econometrics test (2015-09-15)

Name: _____

Below, you'll find three exercises; number 1 is obligatory. Then, you have the choice between number 2 and number 3.

1. Say if the following statements are unambiguously true (TRUE), unambiguously false (FALSE) or impossible to classify the way they are stated (CAN'T SAY). Write the motivations to your answers only in the space provided. Answers with no motivations will not be considered.

   (a) An idempotent matrix is invertible.
   TRUE ◯ FALSE ◯ CAN'T SAY ◯

   (b) The estimators $\bar{X}_1$ and $\bar{X}_2$ are consistent for μ. Then $\beta\bar{X}_1 + (1-\beta)\bar{X}_2 \xrightarrow{\text{p}} \mu$ only if $\beta = 0$ or $\beta = 1$.
   TRUE ◯ FALSE ◯ CAN'T SAY ◯

   (c) In the Hausman test, one of the two estimators compared has to be consistent under both H_0 and H_1.
   TRUE ◯ FALSE ◯ CAN'T SAY ◯

   (d) The null hypothesis of the DF test is the stationarity of the time series.
   TRUE ◯ FALSE ◯ CAN'T SAY ◯

   (e) Neglected heteroskedasticity provokes inconsistency in the probit model.
   TRUE ◯ FALSE ◯ CAN'T SAY ◯

2. Let $\mathbf{x}_1$ be a vector of variables, let x_2 be a continuous variables, and let x_3 be a dummy variable. y is a the binary dependent variable.

   (a) In the logit model

   $$P(y = 1|\mathbf{x}_1, x_2, x_3) = \Lambda(\mathbf{x}_1\boldsymbol{\beta}_1 + \beta_2 x_2 + \beta_3 x_3), \tag{1}$$

   find the partial effects of x_2 and x_3 on the response probability. How would you estimate these partial effects?

   (b) The binary dependent variable y is equal to 1 if a man was arrested at least once during 1986 in the US, and 0 otherwise. The explanatory variables are $durat$ which measures the number of months of recent unemployment duration, $income$ is income (in thousand \$), $black$ is a dummy equal to 1 if the individual is black, $hispan$ is a dummy equal to 1 if the individual is Hispanic. Individual with $black = 0$ and $hispan = 0$ are white. The estimation of a logit model of y, with explanatory variables $durat$, $income$, $black$, and $hispan$, returns the estimated parameters reported in Table 1.

	Coefficient	Std. Error	z	p-value
const	−0.939793	0.0792563	−11.8576	0.0000
durat	0.0200870	0.00936375	2.1452	0.0319
income	−0.0659921	0.00883824	−7.4667	0.0000
black	0.766025	0.115345	6.6412	0.0000
hispan	0.448444	0.107716	4.1632	0.0000

Mean dependent var	0.277064	S.D. dependent var	0.447631
McFadden R^2	0.050652	Adjusted R^2	0.047543
Log-likelihood	−1526.725	Akaike criterion	3063.451
Schwarz criterion	3093.002	Hannan–Quinn	3074.132

Number of cases 'correctly predicted' = 1985 (72.8 percent)
Likelihood ratio test: $\chi^2(4)$ = 162.917 [0.0000]

Table 1: Probability of being arrested in 1986 (logit model, 2725 individuals)

   Provide an economic interpretation to the estimated parameters of the variables $durat$, $income$, $black$, and $hispan$.

   (c) On the basis of the estimation results reported in Table 1, compute the probabilities of having been arrested at least once for a "typical" white man, a "typical" black man, and a "typical" Hispanic if $durat = 0$. Motivate your choices on the basis of Table 2.

	White	Black	Hispanic	**Total**
n	1693	439	593	2725
Mean	6.0726	3.2628	5.5064	5.4967
Median	3.5800	0.57000	3.1300	2.9000
Minimum	0.0000	0.0000	0.0000	0.0000
Maximum	50.440	36.910	54.100	54.100

Table 2: Descriptive statistics on income

3. Suppose that the direct taxation system works as follows (y_t is annual income, for which we assume $\Delta y_t = \mu + \varepsilon_t$, with $\mu > 0$):

   - on year t, in November, taxpayers pay an advance, which is a fraction of their income from previous year: $a_t = \gamma y_{t-1}$;
   - on year $t+1$, in May, total taxes are calculated by applying the rate ϕ on the income of the previous year, plus a fixed adjustment u_{t+1};
   - the total balance due at $t+1$ is obtained by subtracting a_t from the quantity calculated above.

   Indicate the time series of tax revenues as x_t; under the assumption

$$\begin{bmatrix} \varepsilon_t \\ u_t \end{bmatrix} \sim WN \left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \right),$$

   (a) Write the system $\begin{bmatrix} x_t & y_t \end{bmatrix}'$ using matrix notation (as a VAR model).

$$\begin{bmatrix} x_t \\ y_t \end{bmatrix} =$$

   (b) Prove that x_t is nonstationary.

   (c) Calculate $E(\Delta x_t)$.

$E(\Delta x_t) =$ _____

   (d) Time series x_t and y_t are clearly cointegrated. Given the answer you provide in (a), write the VECM model.

$$\begin{bmatrix} \Delta x_t \\ \Delta y_t \end{bmatrix} =$$

   (e) Write the second component of the normalised cointegrating vector β.

$$\begin{bmatrix} \beta_1 \\ \beta_2 \end{bmatrix} = \begin{bmatrix} 1 \\ \\ \end{bmatrix}$$