PhD in Economics (17th Cycle)
Econometrics test (2017-05-08)

Name: ______________________________

1. Say if the following statements are unambiguously true (TRUE), unambiguously false (FALSE) or impossible to classify the way they are stated (CAN'T SAY). Write the motivations to your answers only in the space provided. A CAN'T SAY answer with no motivations will be considered wrong.

   (a) Consider two n-dimensonal vectors y and x; the R^2 of the regression of y on x equals the R^2 of the regression of x on y.

   TRUE ○ FALSE ○ CAN'T SAY ○

   (b) Suppose $X_1, X_2, \ldots, X_n$ is a random sample of size n drawn from a distribution with pdf $p(x_i) = \exp\{-\lambda\}\lambda^{x_i}/x_i!$; the sample variance is the maximum likelihood estimator for the unknown parameter λ.

   TRUE ○ FALSE ○ CAN'T SAY ○

   (c) Consider the linear model $y_i = \beta x_i + v_i$, with $v_i = \alpha z_i + \varepsilon_i$, $\mathrm{V}(x_i) < \infty$, $\mathrm{COV}(x_i, z_i) > 0$, and $\varepsilon_i \perp\!\!\!\perp x_i, z_i$. Then $\hat{\beta}_{OLS} \xrightarrow{\mathrm{p}} \theta > \beta$.

   TRUE ○ FALSE ○ CAN'T SAY ○

   (d) Consider the sample selection model $y_i = \alpha + \beta x_i + \varepsilon_i$ where y_i is observed if $z_i^* = \gamma + u_i > 0$ and $\varepsilon_i, u_i | x_i$ are standard normal r.v., with $\mathrm{E}(\varepsilon_i u_i) = \rho$. Then $\hat{\beta}_{OLS}$ is consistent for β.

   TRUE ○ FALSE ○ CAN'T SAY ○

   (e) The covariance matrix of a multivariate white noise is diagonal.

   TRUE ○ FALSE ○ CAN'T SAY ○

2. Consider a sample of N individuals. Assume that $\mathbf{y}$ is the $N \times 1$ binary dependent variable and $d1, d2, \ldots, dM$ are the $N \times 1$ dummy regressors for exhaustive and exclusive categories, meaning that each person in the population falls into one and only one category (if for individual i $dk_i = 1$, all the other $M - 1$ regressors of individual i are equal to 0). Consider the linear regression, without the intercept,

$$\mathbf{y} \text{ on } \mathbf{X}, \text{ with } \mathbf{X} = [d1\ d2\ \ldots\ dM].$$

Prove that:

(a) The matrix $\mathbf{X}'\mathbf{X}$ is diagonal.

(b) The OLS estimates of the coefficients are all in the $[0, 1]$ interval ($0 \leq \hat{\beta}_j \leq 1$, for $j = 1 \ldots M$).

(c) The fitted values from the OLS regression are in the $[0, 1]$ interval for all individuals ($0 \leq \hat{y}_i \leq 1$, for $i = 1 \ldots N$)

3. Figure 1 depicts four quarterly time series, observed in the 1999:1 – 2016:4 interval. The description of the series is as follows:

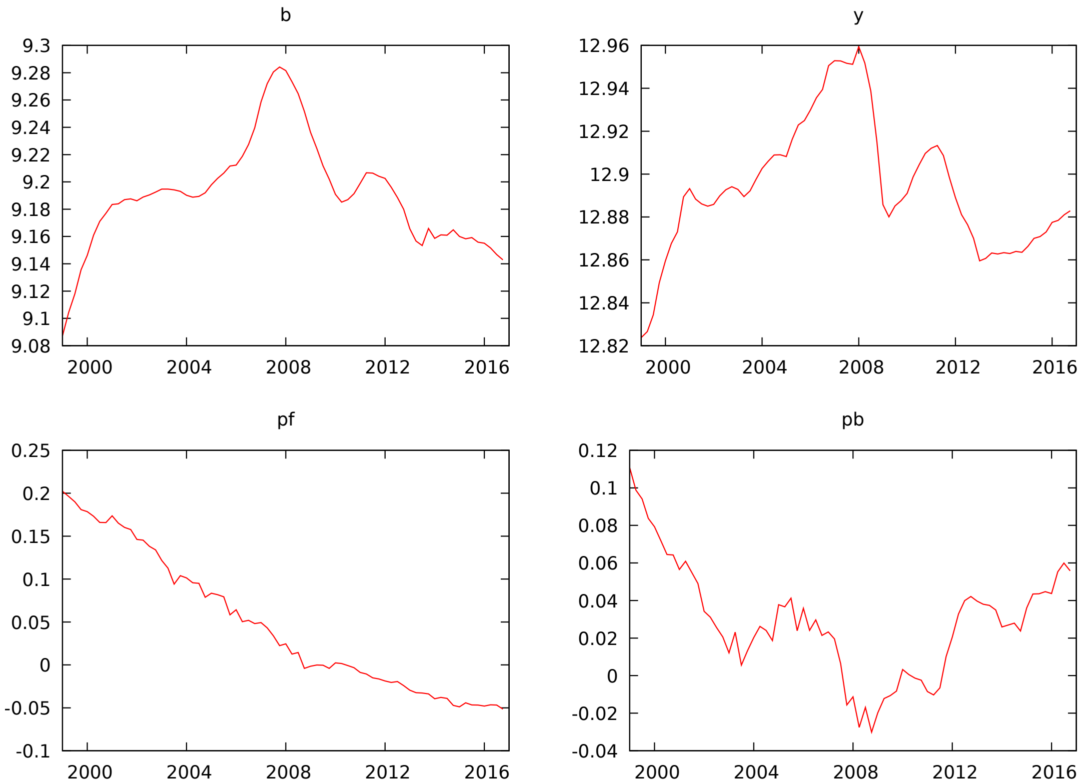

Figure 1: The data

Name	**Description**
b_t	Real expenditure in alcoholic beverages, tobacco and narcotics
y_t	Real GDP
pb_t	Price index for alcoholic beverages, tobacco and narcotics
pf_t	Price index for recreation and culture

	Constant only		With trend	
Variable	Test	p-value	Test	p-value
b	0.153	> 0.1	0.153	0.047
y	0.147	> 0.1	0.145	0.057
pf	0.533	0.039	0.155	0.046
pb	0.218	> 0.1	0.156	0.046

Table 1: KPSS tests

All variables are in logs and seasonally adjusted. KPSS tests were run on all four variables, with the results shown in Table 1. Estimation of an ECM model yielded the results shown in Table 2. A restricted version of the model was also estimated, using the variables $w_t = b_t - y_t$ and $r_t = pb_t - pf_t$. The results are shown in Table 3.

OLS, using observations 1999:3–2016:4 (T = 70)
Dependent variable: Δb_t

	Coefficient	Std. Error	t-ratio	p-value
const	-0.7238	0.3286	-2.2025	0.0314
Δy_t	0.2068	0.0824	2.5108	0.0147
b_{t-1}	-0.1374	0.0385	-3.5714	0.0007
pb_{t-1}	-0.0175	0.0348	-0.5027	0.6170
pf_{t-1}	0.0087	0.0080	1.0819	0.2835
y_{t-1}	0.1541	0.0418	3.6895	0.0005
Δb_{t-1}	0.3711	0.1123	3.3035	0.0016
Δb_{t-2}	0.2103	0.1067	1.9701	0.0533

Mean dependent var	0.000550	S.D. dependent var	0.007684
Sum squared resid	0.000975	S.E. of regression	0.003966
R^2	0.760619	Adjusted R^2	0.733592
$F(7, 62)$	28.14299	P-value(F)	5.39e–17
Log-likelihood	292.0204	Akaike criterion	−568.0407
Schwarz criterion	−550.0528	Hannan–Quinn	−560.8957

LM test for autocorrelation up to order 4 –
Test statistic: LMF = 0.404753, p-value = 0.804461

Table 2: Unrestricted ECM model

(a) Comment on the unit-root tests shown in Table 1.

(b) Compute the long-run multipliers with respect to y_t, pb_t and pf_t for the unrestricted model (Table 2) and comment on the economic meaning of their sign and magnitude.

(c) Perform a test for the validity of the restrictions implied by the restricted version of the model.

(d) Suggest an economic interpretation for the above restrictions and compute the long-run multipliers with respect to y_t, pb_t and pf_t for the restricted model (Table 3).

(e) Suggest alternative approaches to the estimation of the model which exploit the possibility of cointegration.

OLS, using observations 1999:3–2016:4 (T = 70)
Dependent variable: Δb_t

	Coefficient	Std. Error	t-ratio	p-value
const	-0.4661	0.1267	-3.6794	0.0005
Δy_t	0.1781	0.0787	2.2619	0.0271
r_{t-1}	-0.0098	0.0079	-1.2459	0.2173
w_{t-1}	-0.1258	0.0342	-3.6764	0.0005
Δb_{t-1}	0.3855	0.1101	3.5026	0.0008
Δb_{t-2}	0.2031	0.1000	2.0306	0.0465

Mean dependent var	0.000550	S.D. dependent var	0.007684
Sum squared resid	0.000999	S.E. of regression	0.003952
R^2	0.754695	Adjusted R^2	0.735531
$F(5, 64)$	39.38004	P-value(F)	2.84e–18
Log-likelihood	291.1649	Akaike criterion	−570.3298
Schwarz criterion	−556.8388	Hannan–Quinn	−564.9710

LM test for autocorrelation up to order 4 –
Test statistic: LMF = 0.548547, p-value = 0.700763

Table 3: Restricted ECM model