PhD in Economics
1st year Econometrics exam (2018-01-28)
Statistical theory

Please try to write as legibly as you can (in your own interest).

Name: ______________________________

1. Say if the following statements are unambiguously true (True), unambiguously false (False) or impossible to classify the way they are stated (Not necessarily). Write the motivations to your answers **only** in the space provided. A "Not necessarily" answer with no motivations will be considered wrong.

   (a) If X is a discrete variable, its distribution function is differentiable in all $\mathbb{R}$.

   True ◯ False ◯ Not necessarily ◯

   (b) Suppose you have a nondegenerate random variables X with support over the positive reals. Then $E(X^3) > E(X)^3$

   True ◯ False ◯ Not necessarily ◯

   (c) An estimator may be asymptotically normal and yet not consistent.

   True ◯ False ◯ Not necessarily ◯

   (d) GMM estimators may not be consistent.

   True ◯ False ◯ Not necessarily ◯

   (e) Conditional normality of $y_i|\mathbf{x}_i$ is essential for the asymptotic normality of the OLS estimator $\hat{\beta} = [\sum \mathbf{x}_i\mathbf{x}_i']^{-1} \sum \mathbf{x}_i y_i$

   True ◯ False ◯ Not necessarily ◯

2. Consider an iid sample of bivariate Bernoulli random variables (Y, X) where $0 < P(X = 1) = p < 1$, $0 < P(Y = 1) = q < 1$ and $P(Y = 1, X = 1) = \theta \cdot p \cdot q$.

   (a) Prove that, if $p + q = 1$ and $\theta = 0$, then

$$\operatorname{corr}(X, Y) = -1;$$

   (b) prove that

$$n^{-1} \sum_{i=1}^{n} x_i \xrightarrow{\text{p}} p;$$

   (c) check if the statistic

$$\hat{\theta} = n \cdot \frac{\sum_{i=1}^{n} x_i y_i}{\sum_{i=1}^{n} x_i \sum_{i=1}^{n} y_i}$$

   is a consistent estimator for θ;

   (d) prove that, given the linear regression

$$y_i = \beta_0 + \beta_1 x_i + \varepsilon_i \tag{1}$$

   the OLS coefficient $\hat{\beta}_1$ converges in probability to $\frac{q(\theta-1)}{1-p}$.

Answer the points above on a separate sheet. Now suppose that you observe a sample with the following data:

$$n = 1500 \quad \bar{y} = 0.4 \quad \bar{x} = 0.8 \quad \frac{1}{n} \sum x_i y_i = 0.3$$

   (e) Compute the OLS coefficients:

   $\hat{\beta}_0 =$ ________ $\hat{\beta}_1 =$ ________

   (f) Compute the variance estimator

   $\hat{\sigma}^2 =$ ________

   (g) Test the hypothesis $H_0 : \theta = 1$.

   Test type: ________ Distribution: ________ Test statistic: ________

   Decision: ○ Reject ○ Don't reject