PhD in Economics (XXIIIth cycle)

Time Series Econometrics test (2021-01-07)

1. Let the equations for two stochastic processes be y_t and x_t be

$$\begin{cases} y_t = \mu + \phi_1 y_{t-1} + \phi_2 y_{t-2} + \varepsilon_t \\ x_t = \beta y_t + \gamma x_{t-1} + \eta_t, \end{cases} \qquad \text{with} \qquad \begin{bmatrix} \varepsilon_t \\ \eta_t \end{bmatrix} \sim MWN\left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} \sigma_\varepsilon^2 & \sigma_{\varepsilon\eta} \\ \sigma_{\varepsilon\eta} & \sigma_\eta^2 \end{bmatrix} \right).$$

Suppose that $|\phi_1| < 1$, $|\phi_2| < 1$, $0 < \gamma < 1$ e $\sigma_{\varepsilon\eta} \neq 0$.

(a) Substitute y_t inside the second equation, write the equation for x_t, and determine which dynamic model it refers to.

$x_t =$ ☐

AR(☐) ARMA(☐, ☐) ADL(☐, ☐)

(b) Calculate the following expectation.

$E(y_t \eta_t) =$ ☐

Suppose your goal is to make an OLS estimate of the above equation of x_t, the result just obtained implies that

- ☐ the estimator of the parameters vector is unbiased and consistent
- ☐ the estimator of the parameters vector is unbiased and efficient
- ☐ the estimator of the parameters vector is biased and consistent
- ☐ the estimator of the parameters vector is inconsistent
- ☐ one of the Gauss-Markov hypotheses has been violated
- ☐ the OLS estimation does not return a closed-form solution (numerical methods are required)
- ☐ the process y_t is nonstationary
- ☐ the process x_t is stationary

(c) If the second equation above was $x_t = \beta y_{t-1} + \gamma x_{t-1} + \eta_t$ would the answers given in (b) have been the same?

☐ YES ☐ NO ☐ CAN'T SAY

Why?

☐

(d) A VAR(2) model containing the equations $y_t = \mu + \phi_1 y_{t-1} + \phi_2 y_{t-2} + \varepsilon_t$ e $x_t = \beta y_{t-1} + \gamma x_{t-1} + \eta_t$ is given by

$$\begin{bmatrix} y_t \\ x_t \end{bmatrix} = \begin{bmatrix} \mu \\ 0 \end{bmatrix} + \begin{bmatrix} \phi_1 & 0 \\ \beta & \gamma \end{bmatrix} \begin{bmatrix} y_{t-1} \\ x_{t-1} \end{bmatrix} + \begin{bmatrix} \phi_2 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} y_{t-2} \\ x_{t-2} \end{bmatrix} + \begin{bmatrix} \varepsilon_t \\ \eta_t \end{bmatrix}.$$

Choose and complete one of the following statements.

☐ the VAR(2) is stationary if the following condition is met: ☐ ;

☐ it is not possible to determine if the VAR(2) is stationary, as the following information is missing:

1. ☐
2. ☐
3. ☐

2. With reference to the previous exercise, let y_t be the first difference of the Euro OverNight Index Average (`eonia` rate on interbank lendings), while x_t is the first difference of the trading volume on the interbank market. Using a sample of $T = 2703$ daily observations, the following estimation output is available:

$$\underset{}{\texttt{vol}_t} = \underset{(0.0002)}{-0.0005} + \underset{(0.0159)}{0.5740}\texttt{vol}_{t-1} - \underset{(0.0699)}{0.6880}\texttt{eonia}_t + \underset{(0.0856)}{0.8020}\texttt{eonia}_{t-1} - \underset{(0.0697)}{0.1654}\texttt{eonia}_{t-2},$$

where standard errors are indicated in parentheses.

(a) Carry out a proper test for the null hypothesis $H_0 : \gamma = 1$.

Test: ☐ Distribution: ☐ Test statistic: ☐

Decisione: DON'T REJECT ☐ REJECT ☐

(b) Write the estimated model using the polynomials in the lag operator.

(☐)$\texttt{vol}_t$ = const + (☐)$\texttt{eonia}_t$

(c) Is the estimated process $\texttt{vol}_t$ stationary?

☐ YES ☐ NO ☐ CAN'T SAY

why?

☐

(d) Calculate the values of the following multipliers:

impact multiplier = ☐ long-run multiplier = ☐