Econometrics test

PhD in Economics (XXIVth cycle)

2022, January 12th — 3 hours

Name: ______________________________ Surname: ______________________________

1. Consider an *i.i.d.* sample $x_1 \dots x_n$, where the density function for x_i is

$$f(x_i;\alpha) = \alpha x_i^{\alpha-1},$$

with $x_i \in [0,1]$ and $\alpha > 0$. Answer the following questions:

(a) For $\bar{X} = \frac{1}{n}\sum_{i=1}^{n} x_i$, prove that the statistic

$$\bar{\alpha} = \frac{\bar{X}}{1-\bar{X}}$$

is a consistent estimator of α. *(Hint: find the probability limit of $\bar{X}$)*

Proof:

(b) Find the maximum likelihood estimator $\hat{\alpha}$.

$\hat{\alpha} =$ ______________________________

(c) Construct a Lagrange Multiplier test for the hypothesis $\alpha = 1$.

LM test statistic: ______________________________ $\xrightarrow{\text{d}}$ ____________

(d) Optional: provide a justification for the statement "$\bar{\alpha}$ is asymptotically less efficient than $\hat{\alpha}$".

Proof:

2. **(Esame PhD del 12/01/2022 - Es. 1)** Let the time series model be $\Phi(L)y_t = \mu + \Theta(L)\varepsilon_t$, with $\varepsilon_t \sim WN(0,1)$. A conditional maximum likelihood estimation returned the following results:

$$(1 - 0.5L + 0.04L^2)y_t = 1.35 + (1 + 0.25L^2)\varepsilon_t.$$

(a) Write the solutions for both polynomials $\Phi(L)$ and $\Theta(L)$.

solutions for $\Phi(L) =$ ________________ and solutions for $\Theta(L) =$ ________________

(b) After checking for possible common factors, write the order of the model

ARMA(____, ____)

(c) Is the process stationary?

○ YES ○ NO ○ CAN'T SAY

Why? ____

(d) Is it possible to write the model in an observationally equivalent $MA(\infty)$ representation?

○ YES, the $MA(\infty)$ representation is

$$y_t = \_\_\_ + \_\_\_\varepsilon_t + \_\_\_\varepsilon_{t-1} + \_\_\_\varepsilon_{t-2} + \_\_\_\varepsilon_{t-3} + \ldots$$

○ NO, because ____

(e) Now consider the stochastic process $x_t = 1.35 + (1 + 0.25L^2)\varepsilon_t$. Compute its expectation, variance and the autocorrelation of order 5.

$E(x_t) =$ ____________, $Var(x_t) =$ ____________, $\rho_3 = Corr(x_t, x_{t-3}) =$ ____________

(a) **(Esame PhD del 12/01/2022 - Es. 2)** With reference to the previous exercise, indicate the parameters of the $\Phi(L)$ polynomial with ϕ_1 and ϕ_2, and the parameter of the $\Theta(L)$ polynomial with θ. Write the null hypothesis (H_0) according to which the model is an AR(1).

H_0: ______________________________

(b) Suppose the covariance of the estimated parameters $\hat{\mu}$, $\hat{\phi}_1$, $\hat{\phi}_2$ and $\hat{\theta}$ is $2 \cdot 10^{-4}$ times the identity matrix, carry out the test about the above null hypothesis.

Test:	____________	Distribution:	________	Test statistic:	____________
Decisione:	DON'T REJECT	○	REJECT	○	

(c) Test the null hypothesis (H_0) according to which the above ARMA model would become a random walk $\Delta y_t = \varepsilon_t$.

H_0: ______________________________

Test:	____________	Distribution:	________	Test statistic:	____________
Decisione:	DON'T REJECT	○	REJECT	○	

(d) Now assume $\Theta(L) = 1$. Is it possible to write the model in *companion form*?

○ YES, the equation is

$$\begin{bmatrix} y_t \\ y_{t-1} \end{bmatrix} =$$

○ NO, because ____

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