Econometrics test

PhD in Economics (XXVIth cycle)

2025, January 10th — 3 hours

Name: _____ Surname: _____

Write all the answers on this sheet

1. **(PhD Exam, 2025-01-10 - Es. 1)** The following estimation output is obtained from an dynamic OLS regression. The theoretical model is $y_t = \mu + \phi_1 y_{t-1} + \phi_2 y_{t-2} + \beta_0 x_t + \beta_1 x_{t-1} + \varepsilon_t$, where $\varepsilon_t \sim WN(0, 3)$, the dependent variable is y_t, while x_t and all the lagged variables y_{t-1}, y_{t-2} and x_{t-1} are the regressors.

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Model 1: OLS, using observations 2019-04-12:2025-01-09 (T = 1500)
Dependent variable: y
                                                  estimated covariance matrix
            coeff   std.err   t-stat    p-value   for the coefficients
  ------------------------------------------------  ------------------------------------------
  const      1.0000  0.8155   1.2262  0.2203          0.6651
  y_1        0.6000  0.0246  24.3902  0.0000 ***     -0.0018   0.0006
  y_2       -0.0500  0.0226  -2.2124  0.0270 **      -0.0072  -0.0004   0.0005
  x          0.2000  0.0081       B1  0.0000 ***     -0.0030   0.0000   0.0000   0.0001
  x_1        0.1000      B2  10.6383  0.0000 ***     -0.0029  -0.0001   0.0001   0.0000   0.0001

Sum squared resid    120769.608   S.E. of regression     8.9879
Log-likelihood       -5419.721    R-squared              0.6180
```

(a) Select the right statements from the following.

The estimated model is

○ ADL(1,1) ○ ADL(2,1) ○ ADL(1,2) ○ ADL(2,2)

The estimated impact multiplier is

○ $\hat{\delta}_0 = \dfrac{0.2+0.1}{1-0.6} = 0.75$ ○ $\hat{\delta}_0 = 1$ ○ $\hat{\delta}_0 = 0.6$ ○ $\hat{\delta}_0 = 0.2$

The slope of the estimated long run relationship ($\bar{y} = b\bar{x}$) is

○ $b = \dfrac{0.2+0.1}{1-0.6+0.05} = \dfrac{2}{3}$ ○ $b = \dfrac{1-0.6+0.05}{0.2+0.1} = 1.5$ ○ $b = 0.2$ ○ $b = \dfrac{0.1}{0.6} = \dfrac{1}{6}$

(b) Fill the blanks in the estimation output.

B1 = _____ B2 = _____

(c) Write the null hypothesis under which the above model reduces to an AR(2) model.

$$H_0: \begin{bmatrix} & & & & \\ & & & & \\ & & & & \\ & & & & \end{bmatrix} \begin{bmatrix} \mu \\ \phi_1 \\ \phi_2 \\ \beta_0 \\ \beta_1 \end{bmatrix} = \begin{bmatrix} \\ \\ \\ \end{bmatrix}$$

(d) The estimated parameters of the AR(2) model obtained under the null hypothesis mentioned in the previous point are $\hat{\mu} \approx 13.35$, $\hat{\phi}_1 \approx 0.72$ and $\hat{\phi}_2 \approx -0.10$. The loglikelihood is $\tilde{\ell} = -5713.058$. Carry out a suitable test on H_0.

Test:	_____	Distribution:	_____	Test statistic:	_____
Decision:	DON'T REJECT	◯	REJECT	◯	

(e) Is this $y_t \sim$ AR(2) process stationary?

◯ YES ◯ NO

Why? _____

_____

_____

(f) Is it possible to write the estimated AR(2) in companion form?

◯ YES, the equation is

$$\begin{bmatrix} \quad \\ \quad \end{bmatrix} = \begin{bmatrix} \quad \\ \quad \end{bmatrix} + \begin{bmatrix} \quad & \quad \\ \quad & \quad \end{bmatrix} \begin{bmatrix} \quad \\ \quad \end{bmatrix} + \begin{bmatrix} \quad \\ \quad \end{bmatrix}$$

◯ NO, because _____

_____

_____

◯ CAN'T SAY. The companion form is available only when _____

_____

_____

(g) Compute the following quantities from the estimated AR(2) model.

- unconditional expectation $E(y_t) =$ _____,
- autocorrelation $\rho_1 =$ _____,
- $\lim_{k\to\infty} \rho_k =$ _____.

(h) Carry out the Breusch-Godfrey test for autocorrelation up to order 4. The auxiliary regression returns $R^2 \approx 0.0028$.

Test:	Breusch-Godfrey	Distribution:	_____	Test statistic:	_____
Decision:	DON'T REJECT	◯	REJECT	◯	